Quantum Interference and the Aharonov-Bohm Effect

These counterintuitive effects play important roles in the theory of electromagnetic interactions, in solid-state physics and possibly in the development of new microelectronic devices

by Yoseph Imry and Richard A. Webb

Although quantum theory is about a century old, its capacity to produce counterintuitive insights into the nature of matter remains undiminished. One such surprise began with a provocative experiment proposed by Yakir Aharonov and David Bohm in 1959. Imagine, they said, a magnet shielded in such way that it could not exert a force on another magnet nearby. In other words, no conventional manifestation of the shielded magnet's field could be detected. Yet if a beam of electrons were to travel through the vicinity of this shielded magnet, Aharonov and Bohm predicted, the phase of the electron wave function would change. (In quantum theory an electron can be described sometimes as a wave and sometimes as a particle.)

How could one account for the phase change of an electron wave function? Aharonov and Bohm predicted that this effect was due to a physical entity more fundamental than electric and magnetic fields: a potential, whose rate of change over space and time yields the electric and magnetic fields. After three decades the Aharonov-Bohm effect has been demonstrated conclusively in experiments done on electrons traveling through a vacuum, and in the past four years the effect has been observed in very small conducting wires at low temperatures.

The Aharonov-Bohm effect has had considerable influence on how physicists think about electrodynamics. It has long been known that a positive charge passing close to but in no way touching a stationary, negative charge will nonetheless accelerate and change direction. In order to explain such a phenomenon, known as action at a distance, Michael Faraday proposed in 1846 that charges encounter fields that exert electric and magnetic forces. From what was known at the time, fields described the dynamics of charges completely. When the theories of relativity and quantum mechanics were introduced, the potentials, not the electric and magnetic fields, appeared in the equations of quantum mechanics, and the equations of relativity simplified into a compact mathematical form if the fields were expressed in terms of potentials. The experiments suggested by Aharonov and Bohm revealed the physical significance of potentials: a charged particle that passes close to but in no manner encounters a magnetic or electric field will nonetheless change its dynamics in a subtle but measurable way. The consequence of the Aharonov-Bohm effect is that the potentials, not the fields, act directly on charges.

Physicists have been exploring the effect's broad implications in areas ranging from the quantized Hall effect through superconductivity to superstring theory. In the years ahead the Aharonov-Bohm effect may even have a profound impact in electronics. By the end of the century manufacturers hope to be able to produce silicon chips one centimeter square that contain as many as 100 million components. If that number is to be reached or exceeded, a different set of physical principles must be exploited. The Aharonov-Bohm effect may point a way to such a technology.

A two-slit interference experiment provides an elegant demonstration of the wavelike nature of electrons and provides a basis for understanding the Aharonov-Bohm effect. In such an experiment a particle generator emits a beam of electrons all of which have the same energy. The beam is directed at a plate that absorbs electrons. Two narrow, closely spaced vertical slits pierce the plate. They lie just to the left and right of where the beam impinges. Centered behind the slits is a film that records a bright spot every time an electron hits it. After the particle generator has emitted many electrons, a sequence of light and dark bands parallel to the slits emerges on the film. In the center is a bright band that diffuses out on each side into two dark bands. They in turn are flanked by light bands, and so on [*see illustration on page 59*].

How could the electrons produce this pattern? If electrons acted like bullets, they would ricochet off the slits or pass straight through them. The film would, then, record a concentration of impacts directly behind the slits; there would be relatively few hits off to one side or the other. Clearly such an effect cannot account for the complex pattern that is observed.

A better approach (and one that agrees with the behavior quantum theory ascribes to electrons) would be to assume that the particles behave like waves. The feature that characterizes waves as they travel through space and time is an amplitude that varies periodically from a maximum to a minimum and back again. The instantaneous variation in amplitude and

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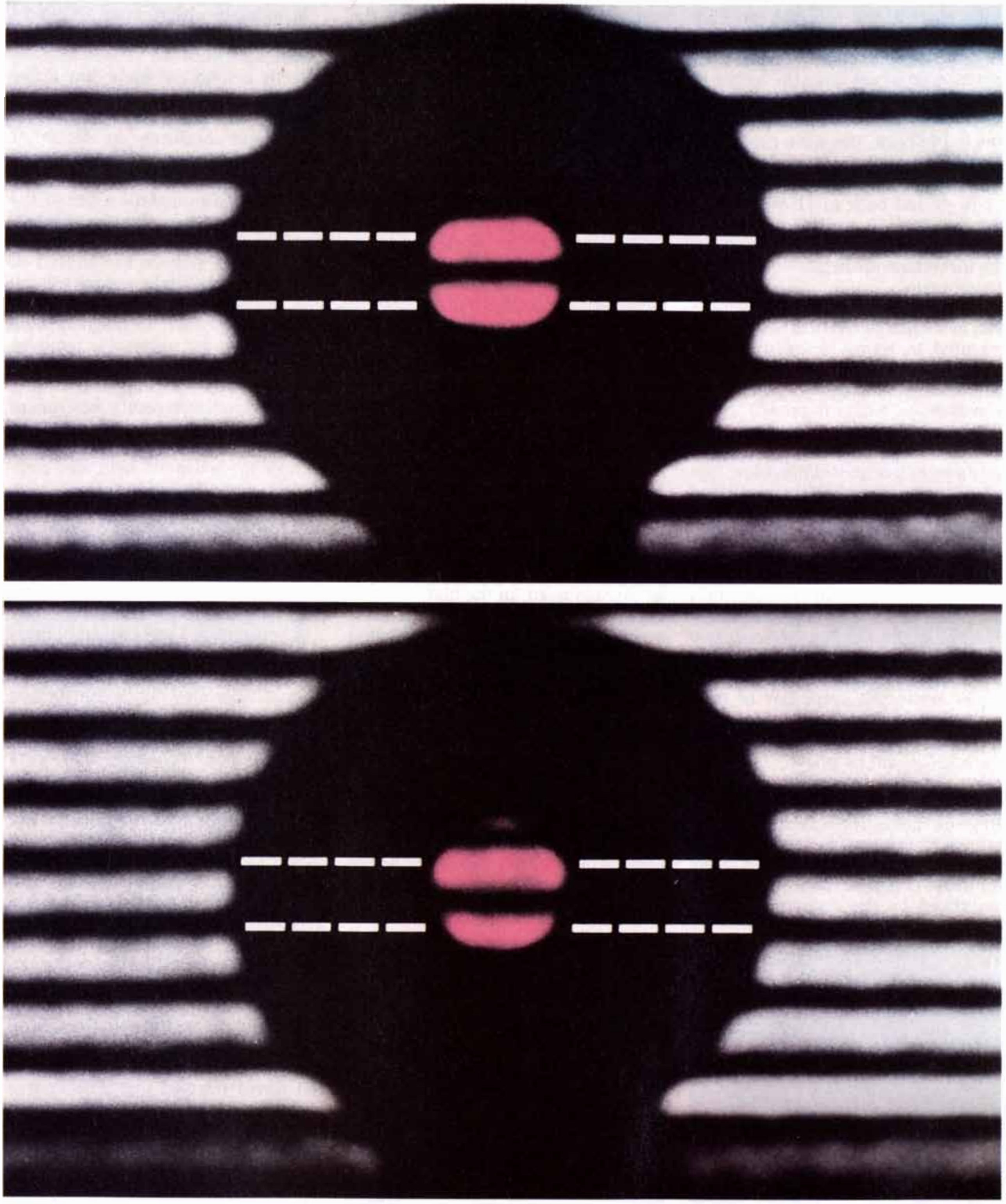


INTERFERENCE PATTERNS demonstrate the Aharonov-Bohm effect. Part of an electron beam passes through a toroidal magnet (*black ring at top*) coated with niobium. The other part of the beam passes outside the toroid. Together the beams cause an interference pattern (*colored region at top*). The background interference pattern outside the ring results from interference among electrons that do not go through the toroid. The interference pattern framed by the toroid is shifted with respect to the background even though the electrons were shielded from the magnetic field. The shielding occurred because the niobium was cooled below 9.1 degrees Kelvin and became superconducting. The shift, as predicted by Yakir Aharonov and David Bohm, is the result of interaction among the electron waves and the vector potential, which is present even in the absence of the magnetic field. When the niobium coating is heated above 9.1 degrees K (*bottom*), it ceases to be superconducting, the magnetic field contained within the toroidal magnet changes and the interference pattern shifts abruptly (*colored region at bottom*). Akira Tonomura and his colleagues carried out this experiment in 1986 at Hitachi Ltd. in Tokyo.

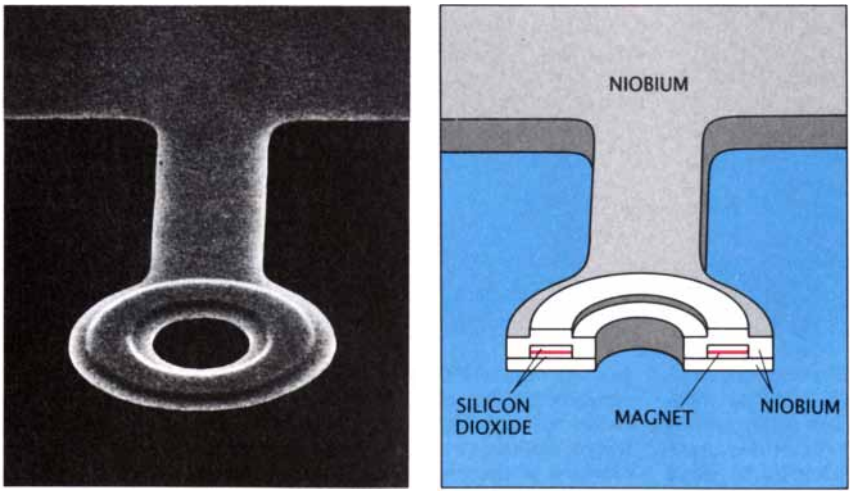
other characteristics of the wave are conveniently described by a mathematical wave function. Consider, for instance, an ocean wave the height of which varies from one meter above the average surface to one meter below it and back. The wave can be described by a cosine function, since the value of the cosine changes from +1 to −1 and back to +1 as its angle changes from 0 to 180 to 360 degrees. The angle that corresponds to the instantaneous height is called the phase angle.

The mathematical wave function that describes an electron wave is represented in terms of its maximum amplitude and phase angle. The amplitude of an electron wave describes a probability, which is related to the fact that the position and velocity of a particle can be determined to within only a certain degree of precision. Specifically, the square of the maximum amplitude of the electron's wave function is the probability of finding the electron at a particular location at a particular time.

The phase angle of an electron's wave function is especially useful for describing the relation between two waves. If two waves are "in phase" at a particular location or time, the two waves are in the same part of their cycle: both have reached maximum or minimum amplitude. If two waves are "completely out of phase," one wave has reached a maximum while the other is at a minimum. The phase angle of an electron wave can also be expressed in terms of more intuitive physical quantities. In simple cases the phase is related to the momentum multiplied by the distance the electron wave has traveled and also to the energy multiplied by the time.

These concepts provide an adequate explanation for the pattern yielded by the two-slit experiment. Since the particle generator emits electrons having the same energy and momentum, the electron wave functions have the same phase at a given distance from the generator—a condition known as coherence. As an electron wave penetrates the two slits, it divides into partial waves. Since partial waves travel the same distance to each slit, the partial wave emerging from the lefthand slit has the same phase as the partial wave emerging from the righthand one. Thus at a point on the film that is equidistant from the two slits, the left and right partial waves will be in phase. Hence the waves reinforce each other and produce a bright band in the middle of the film. It is also fair to say that the bright bands represent the fact that electrons have two times as much chance of striking there as they do at an average point on the film.

To the left of the bright band, however, the right partial wave must travel a greater distance than the left partial wave. Consequently, at some points to the left, the two waves will be completely out of phase and will cancel each other. Hence a dark band will form (because electrons have almost no chance of striking there). At a point still farther to the left on the film, the right wave travels such a distance that it is exactly one full cycle behind the lefthand wave. Once again the waves are in phase and create another bright (high probability) band.



TOROIDAL MAGNET and niobium film employed in the experiments of Tonomura are depicted in the photograph at the left and in the illustration at the right. The magnet, which is five microns across, consists of an alloy containing 83 percent nickel and 17 percent iron. A silicon dioxide coating insulates the magnet from the niobium.

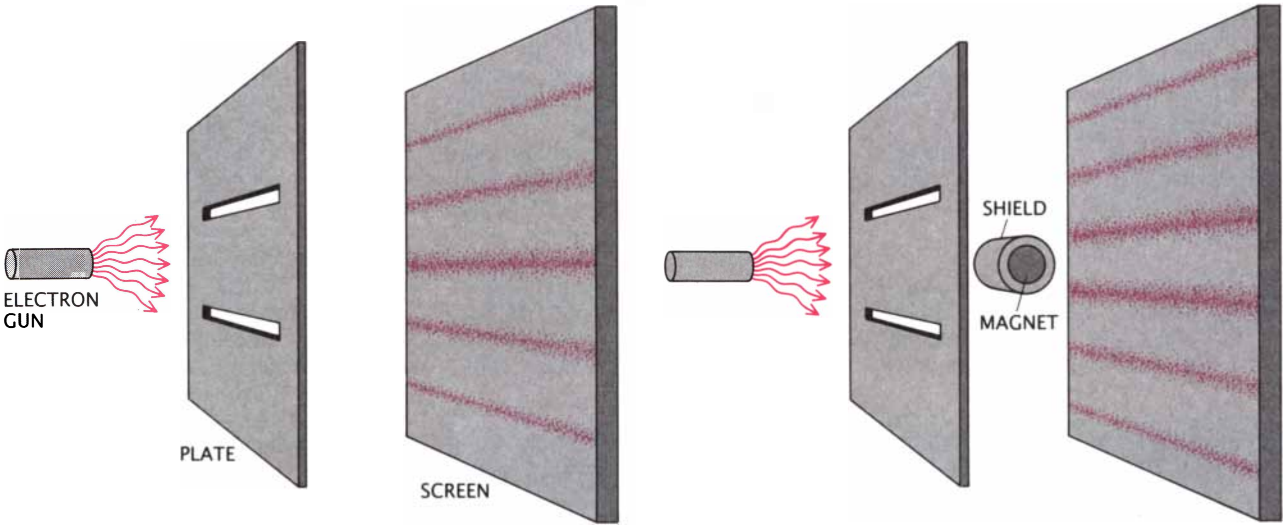
In order to observe the Aharonov-Bohm effect, the two-slit interference experiment must be altered slightly. Directly behind the plate and in between the slits is placed a very long solenoid that has a magnetic field inside it and absolutely no electric or magnetic field outside it. When a beam of electrons now penetrates the two slits and goes around the solenoid, the film records a new interference pattern. Compared with the original pattern, the new pattern has shifted so that previously bright regions will appear darker and dark regions will appear brighter. When the magnetic field contained in the solenoid is removed from the experiment, the interference pattern returns to its original form.

In this new interference experiment the phases of the left and right partial waves apparently changed even though the magnetic field was completely confined inside the solenoid. The change in phase of an electron wave function in a region where no magnetic field exists is one manifestation of the Aharonov-Bohm effect.

The effect revealed that the phase change of a wave function must be related to some physical entity present outside a confined magnetic field. Aharonov and Bohm derived from the fundamental equations of quantum mechanics that the phase change is due to an entity that exists anywhere in and around a magnetic field called the magnetic vector potential. Although the vector potential is a vector field in the sense that it has a magnitude and a direction at every point in space and can change with time, the vector potential can be measured directly only by observing changes in phase of wave functions. The phase shifts caused by the vector potential can account for all measurable magnetic effects on charged particles.

How did the vector potential act on the phase of an electron in the two-slit experiment? As the left and right partial waves traveled in the force-free region near the solenoid, the vector potential changed the momentum of the left partial wave with respect to the right partial wave without changing the kinetic energy. Since the phase of a wave function is related to its momentum, the left partial wave changes phase in relation to the right partial wave.

The magnetic vector potential and the Aharonov-Bohm effect have counterparts in electric interactions. They are the electric scalar potential and the electrostatic Aharonov-Bohm effect. The electric scalar potential is not



TWO-SLIT interference experiment demonstrates the wave behavior of electrons. A particle generator emits a beam of electrons that travels toward two slits in a plate. The electron wave functions pass through the slits, creating an interference pattern (*left*). When a magnet introduces a vector potential field (*right*), the pattern shifts owing to the Aharonov-Bohm effect.

a vector field; it simply has a magnitude at every point in space. Although the absolute magnitude of the potential cannot be determined, the difference in potential between two points is the energy necessary to move a unit of charge from one point to the other. This potential difference is commonly measured in volts.

Like the magnetic vector potential, the electric scalar potential can also cause a phase shift of an electron wave function. The electrostatic Aharonov-Bohm effect is then the phase shift of an electron wave function due to the electric scalar potential in a region where no electric field exists.

The electrostatic Aharonov-Bohm effect can be explained in terms of a thought experiment [*see upper illustration on next page*]. An electron beam is split into two partial waves. Each partial wave is directed into a hollow, metallic cylinder. After the partial waves enter each cylinder, a potential difference is applied between the two cylinders. Before the waves leave, the potential is removed. In this way the waves do not feel an electric force. The total energy difference between the two waves, however, is changed by the charge of the electron multiplied by the difference in potential between the two cylinders. Since the phase of an electron wave function is related to total energy and travel time, the phase of one electron wave is changed with respect to the other.

Soon after Aharonov and Bohm predicted the effects of the potentials on the phase of charged particles, experiments were begun. Robert G. Chambers of the University of Bristol did the first one in 1960. A coherent beam of electrons was generated in an electron microscope and split in two by an aluminum-coated quartz fiber 1.5 microns in diameter. An interference pattern resulted that resembled the pattern produced in the two-slit experiment. When a magnetized iron filament one micron in diameter was placed directly behind the quartz fiber, the pattern shifted. Chambers argued that the magnetic field produced by the filament in the region where the electrons traveled was much too small to explain the magnitude of the observed shift. He concluded that the vector potential must have caused a change in the phase of the electron wave function as predicted by Aharonov and Bohm.

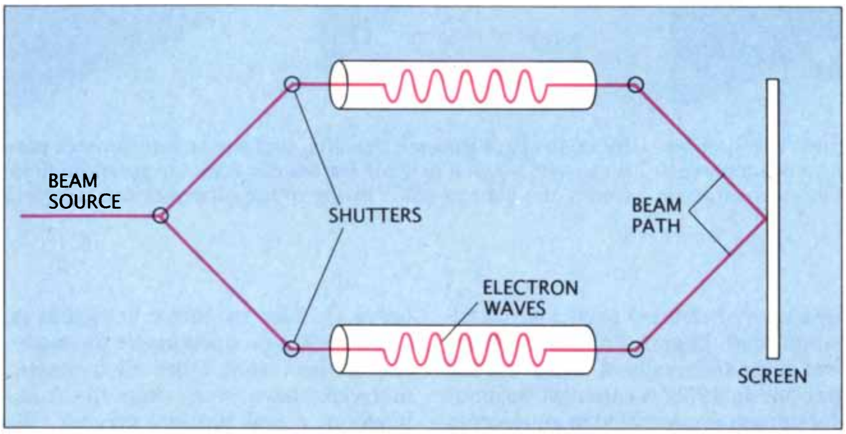
A number of investigators challenged Chambers' conclusion. Since he did not completely confine the magnetic field to a small region of space, some of them maintained that the force exerted by the magnetic field on the electron obscured the contribution of the vector potential to the shift in the interference pattern.

In 1986 Akira Tonomura and his colleagues at Hitachi Ltd. in Tokyo solved the problem of magnetic field confinement, following the suggestion of Charles Kuper of Technion–Israel Institute of Technology. Tonomura knew that a homogeneous magnetic material in the shape of a toroid has a circular magnetic field that is completely contained inside the material of the toroid. Since all magnetic materials have some imperfections, however, a real toroidal magnet will always have some small unconfined magnetic field. This so-called leakage field can be confined by coating the magnet with a superconducting material. Therefore Tonomura constructed a toroidal magnet and coated it with niobium, which superconducts at temperatures below 9.1 degrees Kelvin. This arrangement ensured that less than 1 percent of the field inside the toroidal magnet penetrated into the central hole.

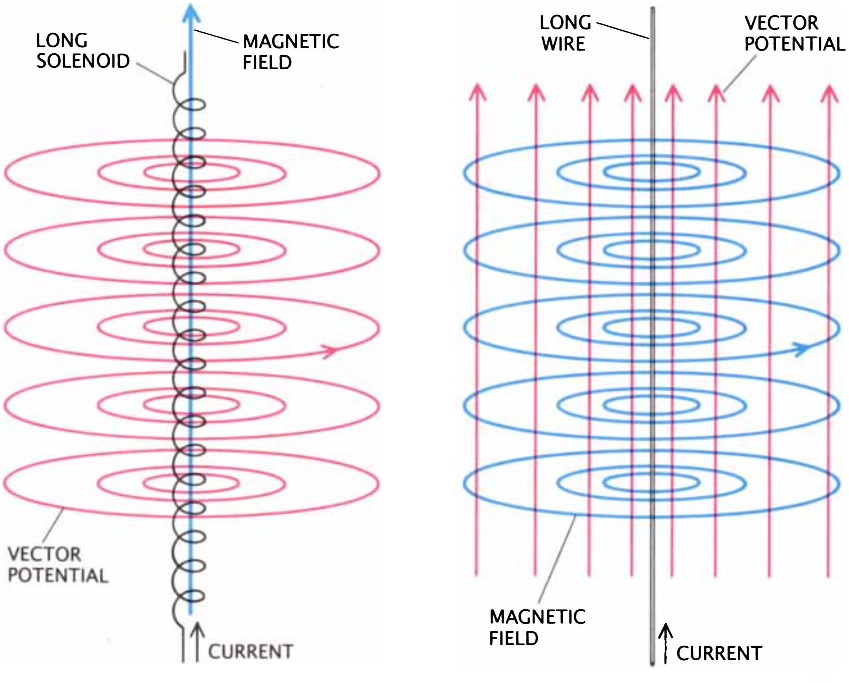
A beam of electrons was generated in a vacuum and split in two so that one beam passed through the hole of the toroid and the other passed outside. Together the beams caused an interference pattern on a film directly behind the hole. A reference interference pattern was created simultaneously. When the niobium was cooled well below 9.1 degrees K and became superconducting, the magnetic field was confined to a specific strength by the laws of superconductivity, and so the vector potential in the hole could attain only a specific strength. As a result the vector potential changed the phase of the electrons traveling through the toroid relative to those traveling outside. This meant that (depending on the particular experimental arrangement) in some cases the interference pattern behind the hole exactly matched the background pattern; in other cases the pattern behind the hole exactly mismatched the reference, that is, the dark bands of one existed alongside the bright bands of the other. Whichever the case, this gave a unique verification of the role of the vector potential in changing the phases of electron waves in a region where no magnetic field exists [*see illustration on page 57*].

When electrons travel through a vacuum, the Aharonov-Bohm effect can be observed because the phase of the electron wave function remains well defined as the wave splits and interferes. The effect is harder to observe in solids, because electrons scatter off various imperfections in the crystal lattice.

Although every solid exhibits some form of scattering, techniques have been developed over the past decade that reduce scattering to the point where electrons travel much as they do through a vacuum [see "Ballistic Electrons in Semiconductors," by Mordehai Heiblum and Lester F. Eastman; SCIENTIFIC AMERICAN, February, 1987].



ELECTROSTATIC Aharonov-Bohm effect can be observed by splitting an electron beam and directing it toward two hollow, metallic cylinders that shield electrons from electric forces. As electrons pass through the cylinders, a scalar potential difference (voltage) is applied between the cylinders. The interference pattern observed on the screen is shifted by an amount directly related to the scalar potential.



VECTOR POTENTIAL FIELD (*red lines*) is compared with the magnetic field (*blue lines*) for a long solenoid (*left*) and a long wire (*right*). Each line represents its respective field at a given strength. The circulation of the vector potential field around a curve is equal to the magnetic field multiplied by the area bound by that curve.

It was, however, the appreciation of two types of scattering in solid conductors—inelastic and elastic—that led to the first discoveries of the Aharonov-Bohm effect and other quantum-interference effects in solid materials.

Inelastic scattering occurs when atoms that make up a solid conductor exchange energy with the electron. Strictly speaking, inelastic scattering alters the wave functions of the atoms making up the solid, that is, scattering induces a change in the quantum state of the environment in which the electron moves. For instance, the electron can absorb energy from or give energy to the vibrations of atoms in a crystal lattice. One key to reducing inelastic scattering is to limit the energy available for such interactions. If enough energy is removed from the crystal lattice and the electron system so that they are essentially quiescent, inelastic scattering will be scarce. The way one can remove this energy is to cool the wire to low temperatures. At quite attainable temperatures of a few degrees Kelvin, electrons in many metals can move across several thousand atoms (a distance of approximately one micron) without undergoing inelastic scattering.

Cooling a solid conductor to low temperatures has another beneficial effect. The range of energies with which electrons travel through a solid decreases as the temperature decreases. At temperatures low enough to make inelastic scattering improbable, the range of energies is so narrowly defined that all electrons traveling through the wire have effectively the same energy. This makes all conducting electrons in the solid interfere in essentially the same way.

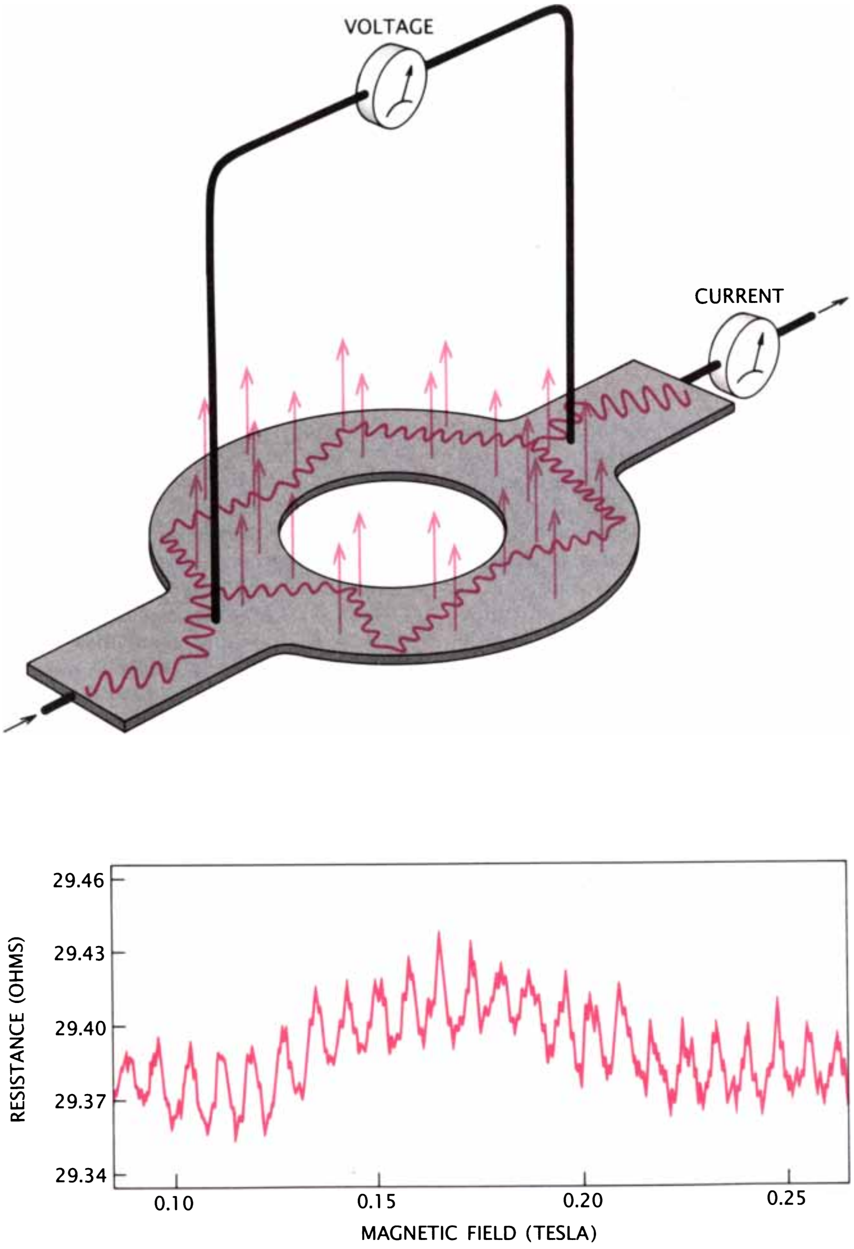
Elastic scattering takes place when an electron encounters a static potential, such as an impurity or a defect in the crystal lattice. A static potential changes the phase of the electron wave function in a well-defined manner but not its total energy. Although a random distribution of static potentials in a solid will lead to a random change in phase, the change will be the same for every electron that travels through the solid at a particular energy. As temperature approaches absolute zero, it turns out that an electron wave should encounter only elastic scattering, which leads to a random but constant phase change and does not obscure electron-interference effects in a solid conductor. That was the key to observing quantum-interference effects in solids.

In real experimental systems, however, solid conductors cooled to low temperatures still exhibit some degree of inelastic scattering that will introduce some uncertainty in the phase of the electron wave function. As the size of the solid conductor decreases, the number of phase-randomizing events decreases. To observe quantum interference the conductor must be sufficiently small to essentially eliminate inelastic scattering. Experiments have shown that, although a metal wire .03 micron thick, .03 micron wide and one micron long contains nearly 100 million atoms, the phase of an electron wave function traveling through the wire will typically be maintained at temperatures below one degree K.

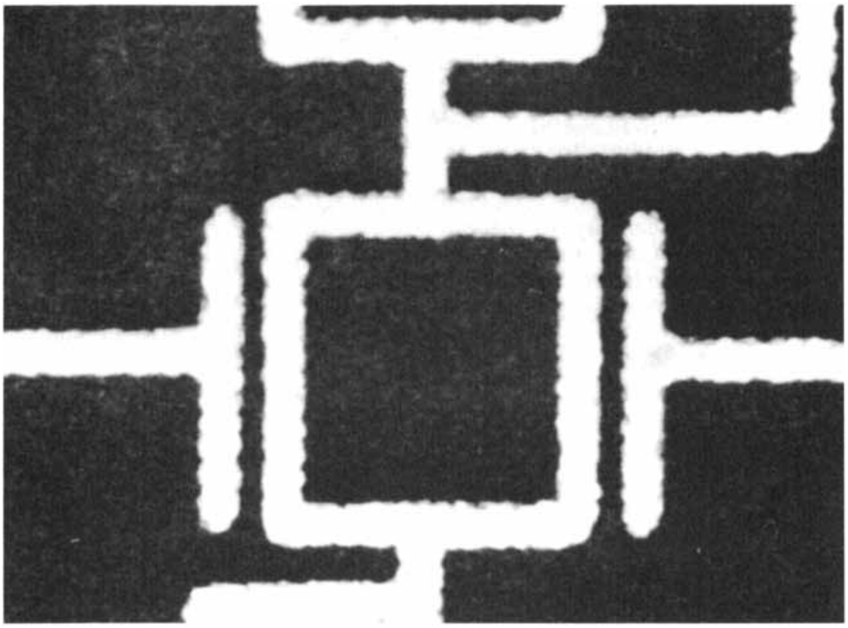
In order to measure electron-interference effects in solid conductors, one must translate the mechanics of electron waves into physical quantities that can be measured easily. When an electron wave travels through a small wire at low temperatures, part of the wave scatters from one end to the other while other parts scatter back to their point of origin. A measure of the difficulty an electron wave has traveling from one end of a wire to the other is electrical resistance; conversely, a measure of the ease with which the wave function moves is the wire's conductance. More than 25 years ago Rolf Landauer of the IBM Thomas J. Watson Research Center in Yorktown Heights, N.Y., developed a theoretical framework expressing the conductance in terms of the probability that an electron wave will be transmitted through the wire. His work shows that the conductance is approximately proportional to the transmission probability divided by a fundamental quantum unit of resistance: 25,812 ohms. This value is equal to Planck's constant divided by the charge of an electron squared.

One factor that contributes to transmission probability and conductance is wave-function interference. Markus Büttiker, Landauer and one of us (Imry) did theoretical work on metallic rings without leads, which demonstrated that elastic scattering did not destroy quantum-interference effects. Then Yuval Gefen, Mark Ya. Azbel and one of us (Imry) predicted in 1984 that, as a result of the Aharonov-Bohm effect, the electrical resistance of a metal ring would oscillate periodically as a magnetic field applied to the center of the ring varied smoothly. When the electron wave functions traveling in two different sections of the ring reinforce each other, the transmission probability and thus the conductance should increase. When the electron wave functions cancel, the transmission probability and the conductance should decrease. Hence the conductance or resistance of a wire should oscillate between these two extremes. In 1981 Boris L. Al'tshuler, Arkady Aronov and Boris Spivak of the Leningrad Institute of Nuclear Physics made a related prediction, and Yuri V. Sharvin and his son at the Institute for Physical Problems in Moscow confirmed it experimentally.

One of us (Webb), working with Sean Washburn, Corwin P. Umbach and Robert B. Laibowitz of the Thomas J. Watson Research Center first demonstrated the Aharonov-Bohm effect in small metallic rings in 1985. The group fabricated a gold ring on a silicon wafer. The ring had an inside diameter of .78 micron and an outside diameter of .86 micron. A current was applied through an input lead attached to one side of the loop and collected at an output lead on the opposite side of



RING measures the Aharonov-Bohm effect in solid conductors (*top*). Electron waves enter from the left and scatter through the ring, which has been cooled to low temperatures. A vector potential field due to a magnetic field (*arrows*) shifts the phase of the electron wave function and changes the ring's electrical resistance, which is determined by measuring the voltage and the current. The Aharonov-Bohm effect accounts for the oscillation in the electrical resistance of the ring (*bottom*).



SWITCHING DEVICE can be based on the electrostatic Aharonov-Bohm effect. An antimony loop .8 micron on a side is flanked by two bars. By applying a potential difference (voltage) to either of the bars or to both, the wave functions of the electrons that travel through the loop change phase, so that the output voltage is altered.

the ring [*see illustration on preceding page*]. Additional wires were attached near the loop to each current lead to measure the voltage drop across the ring. The voltage divided by the current yielded the resistance of the ring. A magnetic field applied perpendicularly created a magnetic vector potential that circulated in the plane of the sample.

The workers observed that the electrical resistance of the ring oscillated periodically as the magnetic field increased. This agreed with what is known about the Aharonov-Bohm effect and potentials. Electron waves that traveled around the gold ring in a clockwise direction interfered with the electron waves that traveled in the opposite direction. As the magnetic field (and the vector potential) was increased, the waves traveling clockwise shifted phase in relation to the waves traveling counterclockwise. As the phase was shifted through a full cycle by the vector potential, the resistance of the ring fluctuated. The average period of oscillation in terms of the magnetic field was .0076 tesla. This quantity multiplied by the average area enclosed by the ring yields a fundamental, quantum-mechanical value equal to Planck's constant divided by the charge of an electron, as predicted theoretically.

The magnitude of the resistance oscillation in this case was quite small: about .1 percent of the total resistance of the ring. Daniel E. Prober of Yale University, Supriyo Datta of Purdue University and their colleagues quickly confirmed the resistance oscillations in other metals and in semiconductors. More recently, experiments by numerous other groups have demonstrated oscillations as large as 50 percent of the total resistance. Furthermore, oscillations in the conductance of these samples are independent of the average resistance and are roughly equal to the charge of an electron squared divided by Planck's constant. Such universality (that is, resistance oscillations independent of the material and its elastic-scattering impurities) was first predicted by Al'tshuler and shortly thereafter by Patrick A. Lee of the Massachusetts Institute of Technology and A. Douglas Stone of Yale University.

The observation of the Aharonov-Bohm effect has opened up an entire new field of research in which the quantum nature of the electron moving in a solid can be studied in the domain that lies between atoms and macroscopic objects. Such "mesoscopic" systems, which are much larger than an atom or a molecule, can be manipulated and measured by macroscopic means, yet they still play by the rules of the game of microscopic physics. These systems directly display the unusual effects of quantum mechanics in, for example, ordinary electrical measurements. It is as though one could measure the resistance of the electrons orbiting an atom. These systems will help to answer fundamental questions such as how large a system must be to behave macroscopically.

Interesting in their own right, the Aharonov-Bohm effect and quantum interference may play a particularly important role in the future of electronics. Since the discovery of the transistor the dimensions of electronic devices have steadily decreased to the point where fewer than 1,000 atoms make up the width of a wire. At the same time the power per unit area that computer chips dissipate in the form of heat has increased. Unless new devices are developed that perform reliably and consume less power, a limit on the number of components per chip will be reached. Ultimately this would limit the operating speed of electronic devices.

Recent research on quantum-interference effects indicates that new electronic devices that dissipate extremely small amounts of power could be developed. An experimental prototype of one such device has already been tested in a low-temperature environment. The device controls resistance and voltage by employing a potential to manipulate the wave characteristics of an electron. In the near future, as the size of electronic components continues to decrease, devices could be constructed that maintain the quantum-mechanical behavior of electrons at much higher temperatures. We find it remarkable that the Aharonov-Bohm effect and other quantum-interference effects, which developed from the abstract foundations of quantum mechanics, have found their way to experiments on down-to-earth samples.

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